

Formalization of Lumo IR v1.0.0-b144b77 (2026-03-26)

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1. Introduction

In this document, we write a Lumo IR, built upon:

- *Algebraic Effects* research done by **Effekt** (Brachthäuser et al. 2020).
- *Linear Resource Calculus*(λ^1) proposed by Perceus (Reinking et al. 2021).
- *Mutable Value Semantics* formalized by **Hylo/Val** (Cordonier et al. 2022; 2021).

2. Reading Typing Rules

We'll use several typing judgments with the following contexts:

- Δ : Variable and its Assigned Types, Borrowed. Must own before use.
- Γ : Variable and its Assigned Types, Owned. Use values exactly once.
- E : Named Capabilities available in this context, implemented in a Bundle.

Judgment forms:

- $\Delta \mid \Gamma; E \vdash^V x \Rightarrow A$: The inferred type of value x is A
- $\Delta \mid \Gamma; E \vdash^V x \Leftarrow A$: The value x must be type-checked against A
- $\Delta \mid \Gamma; E \vdash^C M \Rightarrow \underline{B}$: The inferred type of computation M is \underline{B}
- $\Delta \mid \Gamma; E \vdash^C M \Leftarrow \underline{B}$: The computation M must be type-checked against \underline{B}

Pattern binder notation:

- $\text{bv}(p)$: the set (or ordered list) of variables bound by pattern p .
- $\text{bv}(_) = \emptyset$.
- $\text{LetSlots}(p)$: variables introduced as **let** slots by pattern p .
- $\text{MutSlots}(p)$: variables introduced as **mut** slots by pattern p .

Pattern syntax:

$$p ::= _ \mid i(p_0, \dots, p_n) \mid \text{let } x \mid \text{mut } x$$

3. Tags

Tags are second-class names, used for places such as variant constructors and function names inside bundles. They cannot be used alone. For effects, we assume a tag-extraction operation $\text{EffectTag}(e)$ for every effect e (including instantiated effects after generic monomorphization).

4. Types

Lumo IR uses System F with several extensions: higher-order types (ω), recursive types (μ), and a Call-by-Push-Value style separation between values (A) and computations (\underline{B}).

$$\begin{aligned}
K & ::= * \mid K \rightarrow K \\
A & ::= X \mid i \text{ of } [A_0, A_1, \dots, A_n] \mid A + A \mid \text{thunk } \underline{B} \mid \text{mut } A \\
& \quad \mid \text{forall } [X : K] . A \mid A [A] \mid \mu X . A \\
\underline{B} & ::= \text{produce } A \mid A \rightarrow \underline{B} \mid \Pi_{(i \in I)} (i \times \underline{B}_i)
\end{aligned}$$

4.1. Type Formation

We separate value types ("Type") from computation types ("CType").

$$\begin{aligned}
& \frac{A : \text{Type}}{\text{produce } A : \text{CType}} \\
& \frac{A : \text{Type} \quad \underline{B} : \text{CType}}{(A \rightarrow \underline{B}) : \text{CType}} \\
& \frac{\forall i \in I. (\underline{B}_i : \text{CType})}{\Pi_{(i \in I)} (i \times \underline{B}_i) : \text{CType}}
\end{aligned}$$

bundle values are named in E and treated as computation-level entities.

5. On Bidirectional Typing

We use Pfenning Recipe for our type system with following rules included.

5.1. Variable Synthesis

Variables synthesize their type directly from the owned context.

$$\frac{x : A \in \Gamma}{\Delta \mid \Gamma; E \vdash^V x \Rightarrow A} [\text{VAR-SYNTH}]$$

5.2. Annotation Synthesis

A type annotation upgrades checking into synthesis.

$$\frac{\Delta \mid \Gamma; E \vdash^V v \Leftarrow A}{\Delta \mid \Gamma; E \vdash^V (v : A) \Rightarrow A} [\text{ANN-SYNTH}]$$

5.3. Value Synth-to-Check

Any synthesized value can be consumed in checking mode.

$$\frac{\Delta \mid \Gamma; E \vdash^V v \Rightarrow A}{\Delta \mid \Gamma; E \vdash^V v \Leftarrow A} [\text{V-SYNTH-CHECK}]$$

5.4. Computation Synth-to-Check

Any synthesized computation can be consumed in checking mode.

$$\frac{\Delta \mid \Gamma; E \vdash^C M \Rightarrow \underline{B}}{\Delta \mid \Gamma; E \vdash^C M \Leftarrow \underline{B}} [\text{C-SYNTH-CHECK}]$$

5.5. Forall Elimination

Type application instantiates a polymorphic value and synthesizes the instantiated type.

$$\frac{\Delta \mid \Gamma; E \vdash^V u \Rightarrow \text{forall}[X : K].A}{\frac{T : K}{\Delta \mid \Gamma; E \vdash^V u[T] \Rightarrow A[X := T]}} [\text{FORALL-ELIM-SYNTH}]$$

6. Data

Lumo IR uses a single concept for constructing values, called **data**, which can cover the following real-world practices:

- enum: finite **data** variants
- struct: single **data** variant
- primitives: modeled as infinite-variant **data**.
for example: **data** nat ::= {x | x ∈ ℕ}

6.1. Definition of Data

$$\text{data } T = \Sigma_{(i \in I)} i \text{ of } [A_0, A_1, \dots, A_n]$$

The **of** [...] part can be omitted; if omitted, the tag is treated as nullary.

Examples:

$$\text{data Nat} = \text{Zero} + \text{Succ of } [\text{Nat}]$$

$$\text{data Bool} = \text{False} + \text{True}$$

$$\text{data NatList} = \text{Nil} + \text{Cons of } [\text{Nat}, \text{NatList}]$$

6.2. Rules of Data

6.2.1. Introduction

Before constructing data, we must “own” the parameters — each p_i must be available in Γ_i — and consume contexts sequentially so the constructed data owns its values.

$$\frac{\Delta \mid \Gamma_{i+1}, \dots, \Gamma_n; E \mid \Gamma_i \vdash^V p_i \Leftarrow A_i \quad (0 \leq i \leq n)}{\Delta \mid \Gamma_0, \Gamma_1, \dots, \Gamma_n; E \vdash^V i(p_0, p_1, \dots, p_n) \Leftarrow i \text{ of } [A_0, A_1, \dots, A_n]} [\text{DATA-INTRO}]$$

We can get data T by roll-ing the expression above with type assertion ($e : T$).

6.2.2. Elimination

Next we eliminate unrolled data (a sum of variants). **match** requires an owned variable x , and each branch is checked under freshly owned bindings introduced by the match pattern. For the bindings, we can use **let** x and for mut data, **mut** x .

$$\frac{\forall i \in I. \text{MutSlots}(p_i) = \emptyset \quad \frac{\Delta \mid \Gamma, \text{bv}(p_i); E \vdash^C e_i \Leftarrow \underline{B}}{\Delta \mid \Gamma, x; E \vdash^C \text{match } x\{p_i \mapsto e_i\} \Leftarrow \underline{B}} [\text{DATA-ELIM}]}{x : \text{mut } A \in \Gamma \quad \frac{\Delta \mid \Gamma, \text{bv}(p_i); E \vdash^C e_i \Leftarrow \underline{B}}{\Delta \mid \Gamma, x; E \vdash^C \text{match } x\{p_i \mapsto e_i\} \Leftarrow \underline{B}} [\text{DATA-ELIM-MUT}]}$$

6.2.3. β -reduction and η -expansion

When a pattern matches, only bound variables are substituted. Wildcard `_` drops the matched value and contributes no binding.

$$\begin{aligned} \text{binds}(_, v) &= [] \\ \text{binds}(\mathbf{let} \ x, v) &= [x := v] \\ \text{binds}(\mathbf{mut} \ x, v) &= [x := v] \\ \text{binds}(i(p_0, \dots, p_n), i(v_0, \dots, v_n)) &= \text{binds}(p_0, v_0), \dots, \text{binds}(p_n, v_n) \\ \mathbf{match} \ i(v_0, v_1, \dots, v_n) \{i(p_0, p_1, \dots, p_n) \mapsto e_i\}_{i \in I} & \text{ [DATA-BETA]} \\ \longrightarrow e_{i[\text{binds}(i(p_0, p_1, \dots, p_n), i(v_0, v_1, \dots, v_n))]} & \end{aligned}$$

Reconstructing the same data through `match` yields the original value.

$$\begin{aligned} \mathbf{match} \ v \{i(\mathbf{let} \ x_0, \mathbf{let} \ x_1, \dots, \mathbf{let} \ x_n) \mapsto i(x_0, x_1, \dots, x_n)\}_{i \in I} & \text{ [DATA-ETA]} \\ \longrightarrow v & \end{aligned}$$

7. Recursive Types

Recursive types are introduced by `roll` and eliminated by `unroll`. It is crucial, to represent recursive `data` types.

7.1. Introduction / Elimination

$$\frac{\Delta \mid \Gamma; E \vdash^V v \Leftarrow A[X := \mu X.A]}{\Delta \mid \Gamma; E \vdash^V \mathbf{roll} \ v \Leftarrow \mu X.A} \text{ [MU-INTRO]}$$

$$\frac{\Delta \mid \Gamma; E \vdash^V v \Rightarrow \text{mu } X.A}{\Delta \mid \Gamma; E \vdash^V \mathbf{unroll} \ v \Rightarrow A[X := \text{mu } X.A]} \text{ [MU-ELIM]}$$

7.2. β -reduction

$$\begin{aligned} \mathbf{unroll} \ (\mathbf{roll} \ v) & \text{ [MU-BETA]} \\ \longrightarrow v & \end{aligned}$$

8. Function

Functions are computations transforming input data into output data while having capabilities to utilize.

8.1. User-level Syntax

The user only sees the surface-level syntax:

$$\mathbf{fn} \ f[X_0 : K_0, \dots, X_n : K_n](p_0 : A_0, \dots, p_m : A_m) : \underline{B} / \rho := M$$

where

$$p ::= _ \mid i(p_0, \dots, p_n) \mid \mathbf{let} \ x \mid \mathbf{mut} \ x$$

Also, we have the sibling call syntax:

$$\text{Call} ::= u(\bar{e})$$

$$u ::= f \mid f[\bar{A}]$$

Here each argument position in \bar{e} may be filled by either an ordinary expression argument or an explicit slot argument (**let** x or **mut** x). We keep the metavariable e for both forms in the call rules below.

TODO: Slot semantics and metatheory for let/mut parameters are deferred.

8.1.1. Elaboration of User-level Syntax

The user-level syntax mixes several concerns, so we elaborate it step by step.

8.1.1.1. Let Elaboration

In fact, named **fn** is just an alias for **let**. We can define **fn** without giving a name.

$$\begin{aligned}
 & \mathbf{fn} \ f[X_0 : K_0, \dots, X_n : K_n](\\
 & \quad p_0 : A_0, \dots, p_m : A_m \\
 &) : \underline{B} / \rho := M \\
 & \mathbf{let} \ f = \\
 & \quad \mathbf{produce} \ (\qquad \qquad \qquad [\text{FN-LET-ELAB}] \\
 & \quad \quad \mathbf{fn}[X_0 : K_0, \dots, X_n : K_n](\\
 \rightsquigarrow & \quad \quad p_0 : A_0, \dots, p_m : A_m \\
 & \quad \quad) : \underline{B} / \rho := M) \\
 & \quad) \text{ in } f
 \end{aligned}$$

8.1.1.2. Type Parameters Elaboration

Next we'll elaborate type parameters using **forall**. Sequentially transform $X_i : K_i$ into **forall** $[X_i : K_i]$ syntax.

$$\begin{aligned}
 & \mathbf{fn}[X_0 : K_0, \dots, X_n : K_n](\\
 & \quad p_0 : A_0, \dots, p_m : A_m \\
 &) : \underline{B} := M \\
 & \mathbf{forall}[X_0 : K_0]. \\
 & \quad \vdots \qquad \qquad \qquad [\text{FN-TYPE-PARAM-ELAB}] \\
 & \mathbf{forall}[X_n : K_n]. \\
 \rightsquigarrow & \quad \mathbf{fn}(\\
 & \quad \quad p_0 : A_0, \dots, p_m : A_m \\
 & \quad) : \underline{B} := M
 \end{aligned}$$

8.1.1.3. Spine Elaboration

Now let's elaborate the rest – spine of function – at once.

$$\begin{aligned}
 & \mathbf{fn} \ (p_0 : A_0, \dots, p_m : A_m) : \underline{B} := M \\
 & \quad \mathbf{think} \ (\\
 & \quad \quad \mathbf{lambda} \ (p_0 : A_0). \\
 & \quad \quad \vdots \qquad \qquad \qquad [\text{FN-ELAB-MONO}] \\
 \rightsquigarrow & \quad \quad \mathbf{lambda} \ (p_m : A_m). \\
 & \quad \quad (M : \underline{B}) \\
 & \quad)
 \end{aligned}$$

8.1.1.4. Call Elaboration

$$u(\bar{e}) \quad [\text{CALL-ELAB}] \\ \rightsquigarrow (\text{force } u)(e_0) \cdots (e_n)$$

Note that you can pass **let** x or **mut** x slots.

9. CBPV Instructions

9.1. bundle (Computation-Level Π)

$\Pi_{(i \in I)}(i \times \underline{B}_i)$ is a finite computation-level record keyed by tags.

9.1.1. Bundle Introduction

Check each field and assemble them into one bundle.

$$\frac{\Delta \mid \Gamma_{i+1}, \dots, \Gamma_n; E \mid \Gamma_i \vdash^C M_i \Leftarrow \underline{B}_i \quad (0 \leq i \leq n)}{\Delta \mid \Gamma_0, \Gamma_1, \dots, \Gamma_n; E \vdash^C \text{bundle} \{i_0 \mapsto M_0, i_1 \mapsto M_1, \dots, i_n \mapsto M_n\} \Leftarrow \Pi_{(i \in I)}(i \times \underline{B}_i)} [\text{PI-INTRO}]$$

9.1.2. Bundle Elimination

Projecting a field from a synthesized bundle yields that field type.

$$\frac{\Delta \mid \Gamma; E \vdash^C M \Rightarrow \Pi_{(i \in I)}(i \times \underline{B}_i)}{\frac{j \in I}{\Delta \mid \Gamma; E \vdash^C M.j \Rightarrow \underline{B}_j}} [\text{PI-ELIM}]$$

9.1.3. Beta Reduction

Projecting from a literal bundle returns the corresponding field.

$$\frac{(\text{bundle} \{i_0 \mapsto M_0, i_1 \mapsto M_1, \dots, i_n \mapsto M_n\}).j}{\rightarrow M_j} [\text{PI-BETA}]$$

9.2. Lambda

The function type $A \rightarrow \underline{B}$ is made out of **lambda**.

9.2.1. Introduction

$$\frac{\Delta \mid \Gamma, x : A; E \vdash^C M \Leftarrow \underline{B}}{\Delta \mid \Gamma; E \vdash^C \text{lambda } x.M \Leftarrow A \rightarrow \underline{B}} [\text{LAMBDA-INTRO}]$$

Also you can use slot-bindings here:

TODO: Formal proof obligations for slot-bindings (**let/mut** parameters) will be specified later.

$$\frac{\Delta \mid \Gamma, x : A; E \vdash^C M \Leftarrow \underline{B} \quad \text{assign-count}(M, x) \geq 1}{\Delta \mid \Gamma; E \vdash^C \text{lambda } (\text{let } x).M \Leftarrow A \rightarrow \underline{B}} [\text{LAMBDA-LET-INTRO}]$$

$$\frac{\Delta \mid \Gamma, x : A; E \vdash^C M \Leftarrow \underline{B}}{\Delta \mid \Gamma; E \vdash^C \text{lambda } (\text{mut } x).M \Leftarrow A \rightarrow \underline{B}} [\text{LAMBDA-MUT-INTRO}]$$

9.2.2. Elimination

Application consumes the argument and runs the function computation.

$$\frac{\Delta \mid \Gamma_1; E \vdash^C f \Rightarrow A \rightarrow \underline{B} \quad \Delta \mid \Gamma_2; E \vdash^V v \Leftarrow A}{\Delta \mid \Gamma_1, \Gamma_2; E \vdash^C f(v) \Rightarrow \underline{B}} [\text{LAMBDA-ELIM}]$$

9.2.3. β -reduction and η -expansion

$$\begin{aligned} & (\text{lambda } x.M)(v) \quad [\text{LAMBDA-BETA}] \\ & \quad \rightarrow M[x := v] \\ & \text{lambda } x.f(x) \quad [\text{LAMBDA-ETA}] \\ & \quad \rightarrow f \end{aligned}$$

9.3. Thunk and Force

thunk packages a computation as a value, and **force** restores the computation.

9.3.1. Type Formation

$$\text{thunk } \underline{B} : \text{Type}$$

9.3.2. Typing Rules

$$\frac{\Delta \mid \Gamma; E \vdash^C M \Leftarrow \underline{B}}{\Delta \mid \Gamma; E \vdash^V \text{thunk } M \Leftarrow \text{thunk } \underline{B}} [\text{THUNK-INTRO}]$$

$$\frac{\Delta \mid \Gamma; E \vdash^V v \Rightarrow \text{thunk } \underline{B}}{\Delta \mid \Gamma; E \vdash^C \text{force } v \Rightarrow \underline{B}} [\text{FORCE-ELIM}]$$

9.3.3. β -reduction and η -expansion

$$\begin{aligned} & \text{force } (\text{thunk } M) \quad [\text{THUNK-BETA}] \\ & \quad \rightarrow M \\ & \text{thunk } (\text{force } v) \quad [\text{THUNK-ETA}] \\ & \quad \rightarrow v \end{aligned}$$

9.4. Produce and Let

produce injects a value into the computation type **produce** A .

$$\frac{\Delta \mid \Gamma; E \vdash^V v \Leftarrow A}{\Delta \mid \Gamma; E \vdash^C \text{produce } v \Leftarrow \text{produce } A} [\text{PRODUCE-INTRO}]$$

$$\frac{\Delta \mid \Gamma_1; E \vdash^C M \Rightarrow \text{produce } A \quad \Delta \mid \Gamma_2, x : A; E \vdash^C N \Rightarrow \underline{B}}{\Delta \mid \Gamma_1, \Gamma_2; E \vdash^C \text{let } x = M \text{ in } N \Rightarrow \underline{B}} [\text{PRODUCE-ELIM}]$$

9.4.1. β -reduction

$$\begin{aligned} & \text{let } x = \text{produce } v \text{ in } N \quad [\text{PRODUCE-BETA}] \\ & \quad \rightarrow N[x := v] \end{aligned}$$

10. Algebraic Effects

Algebraic effects are expressed with **perform**_{eff} and interpreted by **handle**_{eff}.

10.1. User-level Syntax

Effects are computations passed naturally.

10.1.1. Effect Definition

You group computations by the namespace, and can utilize same syntax as **fn**.

```

effect Test {
  fn argless() : Ret1
  fn generic[A : *]() : Ret1
}

```

So the effect declaration becomes a bundle of elaborated operation types:

```

effect e {
  fn op0[X0,0 : K0,0, ..., X0,n0 : K0,n0](p0,0 : A0,0, ..., p0,m0 : A0,m0) : B0
  ⋮
  fn opn[Xn,0 : Kn,0, ..., Xn,nn : Kn,nn](pn,0 : An,0, ..., pn,mn : An,mn) : Bn
}

```

$$\begin{aligned}
 & e : \Pi_{(i \in I)} (\\
 & \quad \text{op}_i \times (\\
 & \quad \quad \text{forall}[X_{i,0} : K_{i,0}]. \\
 & \quad \quad \vdots \\
 & \quad \quad \text{forall}[X_{i,n_i} : K_{i,n_i}]. (\\
 & \quad \quad \quad A_{i,0} \rightarrow \\
 & \quad \quad \quad \dots \rightarrow \\
 & \quad \quad \quad A_{i,m_i} \rightarrow \\
 & \quad \quad \quad \underline{B}_i \\
 & \quad \quad) \\
 & \quad) \\
 &)
 \end{aligned}
 \tag{EFFECT-DEF-ELAB}$$

\rightsquigarrow

10.2. Effect Operation Invocation

When we have the operation **eff** in our effect handler context, we can invoke **perform_{eff}**.

$$\frac{\text{eff} : \underline{B} \in E}{\Delta \mid \Gamma; E \vdash^C \text{perform}_{\text{eff}} \Rightarrow \underline{B}} \text{[PERFORM]}$$

10.3. Handlers

Of course you can pass the handler so the context takes care of it.

$$\frac{\Delta \mid \Gamma_1; E \vdash^C M \Leftarrow \underline{B} \quad \Delta \mid \Gamma_2; E, (\text{eff} : \underline{B}) \vdash^C N \Leftarrow \underline{C}}{\Delta \mid \Gamma_1, \Gamma_2; E \vdash^C \text{handle}_{\text{eff}} \text{ with } M \text{ in } N \Leftarrow \underline{C}} \text{[HANDLER-INTRO]}$$

10.4. Operational Equations

$$\text{handle}_{\text{eff}} \text{ with } M \text{ in produce } v \text{ [HANDLE-UNIT]} \\ \longrightarrow \text{produce } v$$
$$\text{handle}_{\text{eff}} \text{ with } M \text{ in perform}_{\text{eff}} \text{ [HANDLE-PERFORM-BETA]} \\ \longrightarrow M$$

11. Meta-Theory Roadmap

Priority lemmas.

11.1. Substitution

Goal: prove substitution while preserving linear/accounting side conditions.

Expected effect: enables modular proofs by replacing variables with well-typed terms safely.

11.2. Preservation

Goal: prove that one-step reduction preserves typing and resource/effect invariants.

Expected effect: guarantees type/resource invariants are maintained by every reduction step.

11.3. Progress

Goal: prove that well-typed closed programs are either terminal forms or can reduce.

Expected effect: rules out stuck well-typed closed programs (except designated terminal forms).

11.4. NbE Soundness

Goal: prove soundness of NbE-based definitional equality.

Expected effect: provides a trustworthy basis for definitional equality checks.

11.5. NbE (Normalization by Evaluation)

TODO: NbE formalization is deferred.

Expected benefits:

- Canonical decision procedure for definitional equality.
- Cleaner convertibility checks in type checking.
- Better foundation for future optimizations (normalization-driven simplification).

This document is the base spec for machine-checked proofs.

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